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A COMPARATIVE STUDY ON THE FORECASTING PERFORMANCE OF TIME-VARYIG COEFFICIENT MODELS. EVIDENCE FROM USD/TRY EXCHANGE RATE

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Abstract. Exchange rate forecasting plays an important role in shaping monetary and fiscal policies, ensuring price stability, managing the balance of payments and facilitating foreign trade. At the microeconomic level, accurate exchange rate forecasts allow businesses to manage the currency risks associated with international transactions, make informed decisions about pricing, production and investments, and guide portfolio diversification strategies for investors. Forecasting contributes to the stability and growth of the Turkish economy by providing valuable insight into future exchange rate movements and assists policymakers, businesses and investors in managing the uncertainties of global financial markets. In this study, out-of-sample forecasts for the USD/TRY exchange rate in 2023 were made using data from January 2000 to December 2022. The linearity of the USD/TRY series was tested using the Harvey and Leybourne (2007) and Harvey et al. (2008) tests, which indicated that the series is nonlinear. The stationarity analysis of the series was conducted using the Sollis (2009) and Enders-Lee (2012) tests, which confirmed that the USD/TRY series is integrated of order one I(1). For out-of-sample forecasting in 2023, ARIMA, ARFIMA, F-ARMA, and TvAR models were applied. Based on the forecast performance criteria of RMSE and MAPE, it was determined that the F-ARMA and TvAR models outperformed the ARIMA and ARFIMA models. When comparing F-ARMA and TvAR, it was found that the forecasts obtained from the TvAR model were more accurate.

Keywords: Exchange Rate, Forecasting, ARIMA, ARFIMA, F-ARMA, Time Varying AR.
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1 Introduction

Forecasting exchange rates holds great importance for a country and businesses, as it assists country and business managers in making informed decisions that can impact their economic performance and competitiveness (Alagidede & Ibrahim, 2017). Exchange rate forecasting for a country provides information to guide monetary policy decisions and anticipate and respond to changes in the global economy. For example, a country's central bank uses exchange rate forecasts to guide decisions related to interest rates, which can directly influence the value of the country's currency. Additionally, exchange rate forecasting allows for predicting the impact of changes in global trade flows or economic conditions on a country's economy. This aids in making decisions regarding foreign trade policy and other economic policies (Salvadore, 2019).

Examining exchange rate forecasting from a business perspective, it plays a significant role in making strategic decisions regarding international trade, investment, and operations. A business engaged in exporting goods uses exchange rate forecasts to predict how changes in the value of a currency will affect the prices of their goods in foreign markets. Similarly, a business operating in multiple countries would naturally want to forecast how changes in currency values will affect their revenues and costs. The resilience of their forecasts is associated with how accurately the exchange rate can be predicted (Yu et al., 2005). Therefore, exchange rate forecasting is an indispensable tool for businesses in determining where to invest and operate. It helps them anticipate and respond to changes that can positively or negatively impact their economic performance and competitiveness in the global economy.

In summary, exchange rate forecasting holds vital value for both countries and businesses, as it assists in predicting and responding to changes that can affect their economic performance and competitiveness in the global economy (Amiti et al., 2014).

The USD/TRY exchange rate is an important economic indicator for Turkey, as it can have a significant impact on various aspects of the country's economy. A strong dollar can make Turkish goods and services more expensive for foreign buyers, which can harm industries reliant on exports. On the other hand, a weaker dollar can make Turkish goods and services more competitive, accelerating export growth (Özcan & Kalafatcılar, 2009).

Furthermore, the USD/TRY exchange rate can directly affect inflation and consumer prices by influencing the costs of imported goods and services. A stronger dollar can increase the cost of imported goods and services, leading to higher inflation and a decrease in purchasing power for Turkish consumers (Civcir and Akçağlayan, 2010). In addition, the USD/TRY exchange rate also affects the Turkish government and businesses with dollar-denominated debt. A stronger dollar can make it more expensive for the government and businesses to repay their dollar-denominated debt, putting pressure on their financial stability (Obstfeld et al., 2010).

In summary, the USD/TRY exchange rate has various effects on the Turkish economy, particularly in terms of foreign trade, inflation, and the stability of the financial sector and government (Kandil et al., 2007).

2 Literature Review

In their study, Güneş and Kaya (2021) forecasted the daily exchange rates of the USD, EUR, and GBP using the ARFIMA and FIAPARCH methods. The results indicated the presence of long memory in volatility for all exchange rates. On the other hand, Aliyev et al. (2022) examined Russia's RTS index using daily data and compared the forecasting performance of the LSTM and ARIMA-GARCH methods. The findings suggested that LSTM, a deep learning method, outperformed the ARIMA-GARCH method.

Canıtez and Savaş (2022) predicted the prices of 10,309 Bitcoins using LSTM and ARIMA-GARCH. In contrast to Aliyev et al. (2022) study, this research showed that the ARIMA-GARCH method yielded better results compared to LSTM. Burucu and Bal (2017) employed the ARIMA method to forecast the annual honey production in Turkey for the year 2013. According to the ARIMA prediction, the honey production in Turkey was estimated to reach 121,216 tons in 2023.

Ameur et al. (2023) compared the performance of various methods including LSTM, RNN, GRU, ANN, CNN, and ARFIMA in forecasting daily BCI, BASI, BPMSI, BLSI, BIMS, and BES indexes. The results indicated that LSTM, DRU, and RNN models were selected as the best forecasting methods among others. Derbentsev et al. (2019) investigated the performance of the BART, ARIMA, and ARFIMA methods in forecasting Bitcoin, Ethereum, and Ripple cryptocurrencies using daily data. The study found that BART outperformed ARIMA and ARFIMA in terms of forecasting accuracy. Deðirmenci and Akay (2017) analyzed BIST100, Gold, Exchange Rates, and Oil variables using daily data with the ARIMA and ARCH models. The results revealed the presence of ARCH effect in all ARIMA models considered, and the most

Table 1: Literature Review

Authors	Area	Compared Models	Conclusion
Chiang and Kahl (1991)	Treasury Bill Rate	Constant Coefficient	TvAR is better
Brown et al. (1997)	UK house prices	CPM, VAR, AR	TvAR is better
Lundbergh et al. (2003)	Simulated Data	AR, STAR	TvAR is better
Karakatsani and Bunn (2008)	Electricity prices	Regime Switching, AR, Linear Regression	TvAR is better
D'Agostino et al. (2011)	Unemployment, inflation, interest rate	RW, SV-VAR, VAR-REC	TvAR is better
Dangl and Halling (2012)	S&P500 index	Bayesian model averaging	TvAR is better
Barnett et al. (2014)	Growth and inflation	RSVAR, ST-VAR etc	TvAR is better
Chan (2017)	Inflation	UC-SVM, RW	TvAR is better
Wang et al. (2017)	Real prices of crude oil	FC-CC	FC-TVP is better
Wei and Zhang (2020)	USA	AR, NTV, RW	TvAR is better

successful forecasting results were obtained for the gold variable. Akay et al. (2019) forecasted the Turkish house price index using monthly data and compared the performance of ARIMA, Random Forest, and Hybrid Random Forest methods. The evaluation showed that the Hybrid model outperformed the other methods in predicting the house price index.

Aktağ and Yiğit (2016) examined the inflation variable using monthly data and compared the performance of Box-Jenkins and Artificial Neural Networks (ANN) methods. The performance evaluation indicated that the ARIMA method outperformed ANN. Kumar (2014) investigated the Indian rupee using daily data and analyzed it using the ARFIMA-FIGARCH and ARFIMA FIAPARCH methods. The findings suggested the presence of long memory in both returns and volatility. Sevli (2019) evaluated the performance of Support Vector Machine, Naïve Bayes, Random Forest, K-Nearest Neighbors, and Logistic Regression methods using the Wisconsin Breast Cancer dataset. The logistic regression method achieved the highest accuracy of 98.24% and was identified as the most successful method.

Özmen et al. (2018) performed a performance evaluation of various classification methods, including DVM, Naïve Bayes, J48, Random Forest, Adaboost, Logistic Regression, Single-Layer Perceptron, Multilayer Perceptron, and Bagging Decision Trees, using the 303-heart disease dataset. The findings indicated that the DVM algorithm provided the best results compared to other methods. Additionally, a literature summary comparing the forecasting performances of TvAR models with other models is presented in Table 1.

3 Methodology

3.1 Harvey Linearity Test

Over the past 50 years, the effects of structural breaks and nonlinearity on time series have been extensively studied in time series econometrics. Based on this foundational knowledge, tests have been developed to determine whether a time series is linear or not, drawing on numerous theoretical studies. These tests can be categorized into two main groups:

1. General specification error tests: Ramsey RESET (1969), McLeod-Li (1983), Keenan (1985), Tsay (1986).

2. Tests specific to certain nonlinear structures: Engle (1982), LR test, Terasvirta (1994), and Hansen (1999).

In the past 15 years, new linearity tests have been developed by Harvey and Leybourne (2007) and Harvey et al. (2008).

To apply the test introduced by Harvey et al. (2008) in the literature, the model used under the assumption that the time series is stationary at the level, or in other words, the time series is integrated of order zero (I(0)), can be represented as follows.

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-1}^2 + \beta_3 y_{t-1}^3 + \sum_{j=1}^{p} \beta_{4,j} \triangle y_{t-j} + \varepsilon_t \quad (1)$$

Here, Δ is the differencing operator, and p represents the lag order. The appropriate lag order, as suggested by Harvey et al. (2008), is recommended to be determined as $p_{max} = int\left(8 * \left(\frac{T}{100}\right)^{0.25}\right)$, where T is the length of the time series. The null and alternative hypotheses of the test are expressed as follows.

$H_{0,I(0)} : \beta_2 = \beta_3 = 0 \Rightarrow W_0$ (The series is linear)

$H_{1,I(1)} : \beta_2 \neq \beta_3 = 0$ (The series is not linear)

The test statistic, on the other hand, is calculated using the $W_0 = T * \left(\frac{RSS_0^r}{RSS_0^u} - 1\right)$ formula. Here, T represents the number of observations, while RSS_0^r and RSS_0^u respectively denote the residual sum of squares obtained from restricted and unrestricted equations.

Under the assumption that the time series is non-stationary in levels, that is, it is integrated of order 1 or (I(1)), the model to be constructed is represented as follows.

$$\triangle y_t = \lambda_1 \Delta y_{t-1} + \lambda_2 (\Delta y_{t-1})^2 + \lambda_3 (\Delta y_{t-1})^3 + \sum_{j=1}^{p} \lambda_{4,j} \triangle y_{t-j} + \varepsilon_t \quad (2)$$

The null and alternative hypotheses for this test equation are expressed as follows.

$H_0 : \lambda_2 = \lambda_3 = 0 \Rightarrow W_1$ (the series is linear)

$H_0 : \lambda_2 \neq \lambda_3 \neq 0$ (the series is not linear)

The test statistic, on the other hand, is calculated using the $W_1 = T * \left(\frac{RSS_1^r}{RSS_1^u} - 1\right)$ formula. Here, T represents the number of observations, while RSS_0^r and RSS_0^u respectively denote the residual sum of squares obtained from the restricted and unrestricted equations.

In cases where the degree of stationarity of the time series is not precisely known, Harvey et al. (2008) developed an alternative test statistic using the previous two test statistics. This test statistic is denoted by the

$$W_\lambda = \{1 - \lambda\} W_0 + \lambda W_1$$

formula. The W_λ test statistic follows a χ^2 distribution with two degrees of freedom. In cases where linearity is not detected in the series, the stationarity of the series should be examined using nonlinear unit root tests.

In the literature of nonlinear unit root tests, there have been approximately 20 different tests encountered in the past 25 years, starting from the work of Leybourne et al. (1998) and Enders-Granger in 1998. Fourier-based unit root tests, developed taking into account the frequency characteristics of the time series, are examined under the title 'Unit Root Tests Based on Fourier Functions' in the literature. However, it is also possible to consider these tests as nonlinear unit root tests. For this purpose, in the empirical part, the presence of a unit root in the USD/TRY series is investigated using the Sollis (2009) test, a nonlinear unit root test, and the Enders and Lee (2012) test, which is based on Fourier functions.

3.2 Unit Root Tests

3.2.1 Sollis (2009) Nonlinear Unit Root Test

Nonlinear unit root tests are based on different functional patterns such as SETAR, MTAR, ESTAR, LSTAR, among others. The Sollis (2009) test is a test developed based on the ESTAR-type function from these patterns. Prior to the Sollis (2009) study, Kapetanios et al. (2003)

developed a unit root test based on the same function. The KSS (2003) test assumed that the tendency of the series to revert to the mean is the same at each point. In other words, this test assumes that negative and positive shocks to the series have the same effect. The main difference between the Sollis (2009) test and the KSS (2003) test is that the Sollis (2009) test allows for a model based on the AESTAR function, which allows for differentiation in the impact of negative and positive shocks on the series. The Sollis (2009) unit root test is expressed by the following equation.

$$\Delta y_t = G(\gamma_1, y_{t-1})\{S_t(\gamma_2, y_{t-1})\rho_1 + (1 - S_t(\gamma_2, y_{t-1}))\rho_2\} y_{t-1} + \sum_{k=1}^{k} k_i \Delta y_{t-1} + \varepsilon_t \tag{3}$$

The terms $G(\gamma_1, y_{t-1})$ and $S_t(\gamma_2, y_{t-1})$ in this equation are defined as follows.

$$G(\gamma_1, y_{t-1}) = 1 - exp\left(-\gamma_1\left(y_{t-1}^2\right)\right), \gamma_1 \geq 0$$

and

$$S_t(\gamma_2, y_{t-1}) = \{1 + exp(-\gamma_2 y_{t-1})\}^{-1}, \gamma_2 \geq 0$$

In the case of zero mean, the presence of a unit root in equation (3) is tested with the following hypothesis.

$$H_0 : \gamma_1 = 0$$

In order to test the null hypothesis, γ_2, ρ_1 and $\rho_2$are not defined, thus a first-order Taylor expansion around $\gamma_1 = 0$ needs to be applied to equation (3). As a result, in order to test the presence of a unit root, the following equations are obtained where all parameters are defined.

$$\Delta y_t = \rho_1\gamma_1 y_{t-1}^3 S_t(\gamma_2, y_{t-1}) + \rho_2\gamma_1 y_{t-1}^3 (1 - S_t(\gamma_2, y_{t-1})) + \eta_t \tag{4}$$

Here, $\eta_t = \varepsilon_t + R_t$. R_t represents the remainder term from the Taylor expansion. If equation (4) is rewritten by interchanging $S_t(\gamma_2, y_{t-1})$ and $S_t^*(\gamma_2, y_{t-1})$=$S_t(\gamma_2, y_{t-1})$-0.5,

$$\Delta y_t = \rho_1^*\gamma_1 y_{t-1}^3 S_t^*(\gamma_2, y_{t-1}) + \rho_2^*\gamma_1 y_{t-1}^3 (1 - S_t^*(\gamma_2, y_{t-1})) + \eta_t \tag{5}$$

is obtained. Here, ρ_1^* and ρ_2^* are linear functions of ρ_1 and ρ_2. Equation (5) allows for the same non-linear pattern as equation (6). After obtaining the first-order Taylor expansion of $S_t^*(\gamma_2, y_{t-1})$ around γ_2=0 in equation (5), we reach the equation

$$\Delta y_t = a(\rho_2^* - \rho_1^*)\gamma_1\gamma_2 y_{t-1}^4 + \rho_2^*\gamma_1 y_{t-1}^3 + \eta_t \tag{6}$$

Since a=1/4, equation (6) can be written as follows.

$$\Delta y_t = \phi_1 y_{t-1}^3 + \phi_2 y_{t-1}^4 + \eta_t \tag{7}$$

Here, $\phi_1 = \rho_1^*\gamma_1$ and $\phi_2 = a(\rho_2^* - \rho_1^*)\gamma_1\gamma_2$. In the case of autocorrelation in equation (7), to eliminate the autocorrelation, equation (7) can be expanded and rewritten as follows.

$$\Delta y_t = \phi_1 y_{t-1}^3 + \phi_2 y_{t-1}^4 + \sum_{i=1}^{k} k_i \Delta y_{t-i} + \eta_t \tag{8}$$

Thus, the null hypothesis expressed as $H_0 : \gamma_1 = 0$ in equation (3) will be expressed as $H_0 : \phi_1 = \phi_2 = 0$. While this hypothesis indicates non-stationarity, the alternative hypothesis represents symmetric or asymmetric ESTAR stationarity. In the case of rejecting the null hypothesis, to decide whether the series exhibits symmetric or asymmetric ESTAR stationarity, the hypothesis $H_0 : \phi_2 = 0$ is tested against the alternative hypothesis $H_1 : \phi_2 \neq 0$. Here, the null hypothesis indicates symmetric ESTAR stationarity, while the alternative hypothesis indicates asymmetric ESTAR stationarity. To test the null hypothesis expressed in equation (8), the F critical values shown in Table 1 of Sollis (2009) article should be used instead of conventional F critical values.

3.2.2 Enders and Lee (2012)

This test is observed to have good size and power properties compared to traditional tests. Enders and Lee (2012) developed a new unit root test of the Dickey-Fuller type using Fourier functions in the deterministic term. The deterministic term denoted by α(t) is a function of time, and the Dickey-Fuller-type regression model is represented by equation (9).

$$y_t = \alpha\,(t) + \rho y_{t-1} + \gamma t + \ \varepsilon_t \tag{9}$$

In Equation (9), α(t) represents a time-dependent deterministic function, while ε_t, represents a stationary error term with variance σ_ε^2. The hypothesis being tested is the absence of unit root (ρ=1). In cases where the structure of the deterministic form, α(t), is unknown, it can lead to biased results in testing the null hypothesis. To overcome this problem, it has been suggested that the unknown functional structure of α(t) can be expressed using Fourier terms as shown below:

$$\alpha\,(t) = \alpha_0 + \sum_{k=1}^{n} (\alpha_k sin\frac{2\pi kt}{T} + \beta_k cos\frac{2\pi kt}{T});\,; \quad n \leq T/2 \tag{10}$$

In Equation (10), n represents the frequency count, T represents the number of observations, and k represents the specific frequency count. If $\alpha 1 = \beta 1 = \cdots = \alpha n = \ \beta n = \ 0$, it indicates that the process is linear and requires the application of a traditional unit root test. On the other hand, if there is a structural break or a nonlinear trend in the series, it implies the presence of at least one Fourier frequency in the data generation process. A notable advantage of the Fourier approach is its global rather than local approximation. Additionally, the size and power properties of this test are better than the Augmented Dickey-Fuller (ADF) test, which is a linear unit root test.

If we treat Equation (10) as a regression equation, including a large value for the frequency count n requires the incorporation of numerous frequency components. This can be problematic when the number of observations in the time series is small, as it reduces the degrees of freedom and may lead to overfitting issues. Therefore, instead of specifying the specific form of α(t), it is more reasonable to select appropriate frequencies to be included in Equation (10). Assuming the use of a single frequency k, the test equation can be expressed as follows:

$$\triangle y_t = \rho y_{t-1} + c_1 + c_2 t + c_3 \text{sin}\left(\frac{2\pi kt}{T}\right) \ + c_4 \text{cos}\left(\frac{2\pi kt}{T}\right) \ + \varepsilon_t \tag{11}$$

In Equation (11), the test statistic obtained from testing the null hypothesis ρ=0 is symbolized as τ_{DF_t}, representing the DF version of the test. As the asymptotic properties of the DF version tests are not different from the asymptotic properties of the LM version tests, it is not preferred to demonstrate a separate asymptotic distribution for this test. Another important aspect of the Enders and Lee (2012) test is that the critical values are only dependent on the frequency count (k) and the sample size (T), without being influenced by Fourier terms or other deterministic terms. The critical values generated for the single frequency equation based on Monte Carlo simulations conducted by Enders and Lee are presented in Table 1a and Table 1b in the respective article, while for the equation with cumulative frequency, they are presented in Table 2a and Table 2b.

The estimation process for the appropriate frequency value k in Equation (11) is carried out using a two-step method shown below.

Stage 1: In the first stage, Equation (11) is estimated using the Ordinary Least Squares (OLS) method for $1 \leq k \leq 5$. The value that minimizes the sum of squared residuals (RSS) is selected.

Stage 2: The presence of a nonlinear form is determined using the classical F-test. For this, the null hypothesis denoted as $c_3 = c_4 = 0$ is tested. If the null hypothesis cannot be rejected, it can be concluded that the data generation process occurs with frequency effects.

Conversely, if the null hypothesis is not rejected, the data generation process will be the same as the traditional DF process. Additionally, if the null hypothesis indicating the absence of frequency effects cannot be rejected, and if the error terms obtained from Equation (11) exhibit autocorrelation, Equation (11) can be expanded with lagged values of $\triangle y_t$, resulting in Equation (12) to account for the autocorrelation.

$$\rho y_{t-1} + c_1 + c_2 t + c_3 \sin\left(\frac{2\pi kt}{T}\right) + c_4 \cos\left(\frac{2\pi kt}{T}\right) + \sum_{i=1}^{p} \beta_i \triangle y_{t-i} + \varepsilon_t \tag{12}$$

If the F-statistic value obtained from the application of Equation (11) or (12) is smaller than the critical value, the null hypothesis of the absence of a linear trend is not rejected. In this case, it is recommended to perform the Augmented Dickey-Fuller (ADF) test to further examine the presence of a unit root. For parameter estimation, the following four models have been used:

3.3 Models

The models shown below were utilized in the study Models.

1. ARIMA
2. ARFIMA
3. Fourier-ARMA: F-ARMA
4. Time varying AR: TvAR

3.3.1 ARIMA(p,d,q) Model

The $ARIMA(p, d, q)$ model, introduced by Box-Jenkins (1976), assumes that a time series with dth-degree stationarity is influenced by its own lag of order p and random shocks of order q. The $ARIMA(p, d, q)$ model is expressed as follows:

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + u_t + \theta_1 u_{t-1} + \theta_2 u_{t-2} + \cdots + \theta_q u_{t-q} \tag{13}$$

The equation (13) can be succinctly expressed as follows.

$$Y_t = \sum_{i=1}^{p} \phi_i Y_{t-i} + u_t + \sum_{j=1}^{q} \theta_j u_{t-j} \tag{14}$$

The forecasting stage of the ARIMA(p,d,q) model is based on the Box-Jenkins approach. The steps of the Box-Jenkins approach can be summarized as follows:

1. Identification
2. Estimation
3. Diagnostic testing

In this context, identification refers to determining the orders of p, d, and q parameters in the $ARIMA(p, d, q)$ model. Once the appropriate model orders are determined for the series, the model needs to be estimated using suitable estimation methods. The parameter estimates obtained from the model should satisfy the theoretical constraints and be statistically significant. Additionally, the error terms derived from the model should be free from autocorrelation and exhibit a pure random process (Asteriou & Hall, 2021).

3.3.2 ARFIMA Model

The degree of integration, which measures the resistance of a time series to shocks, may not always be expressed as an integer. The series being I(0) or I(1) indicates whether the series is resistant or non-resistant to shocks. Especially in financial time series, the responses to shocks can vary. In other words, even though shocks may not be temporary, the series can exhibit resistance to shocks. In such cases, it may take a long time for the series to revert to its mean. Series with this characteristic are referred to as long memory series in the time series literature (Mert & Çağlar, 2019). Granger and Joyeux (1980), Hosking (1981), in their respective studies, proposed alternative models that allow for fractional degrees of integration based on the long memory property of time series. This model is known as the $ARFIMA(p, d, q)$ model in the time series literature. The general structure of the $ARFIMA(p, d, q)$ model is similar to the $ARIMA(p, d, q)$ model, but the parameter d is defined as follows in the relevant articles.

$$\nabla^d = (1-L)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-1)^k L^k = \sum_{k=0}^{\infty} \frac{\Gamma(-d+k)}{\Gamma(-d)\Gamma(k+1)} L^k \tag{15}$$

In Equation (15), $-1/2 < d < 1/2$, and Γ represents the gamma function. As the value of the parameter d increases, the resistance of the series to reverting to the mean in the face of shocks also increases. When $d = 0$, it indicates that the time series is short memory and transforms into an ARMA model. If the value of d is in the range $0 < d < 0.5$, the process is considered long memory. If the value of d is in the range $-0.5 < d < 0$, the process is considered medium memory or hyper-differentiated. When $0 \leq d < 1$, it is assumed that the series is non-stationary but will revert to its mean in the long run.

3.3.3 Fourier ARMA Model

There is a vast and growing body of literature indicating that traditional time series models fail to accurately capture the behavior of numerous important economic variables. The fundamental issue lies in the linearity of conventional time series models, which consequently imply a symmetric adjustment process. Let's reconsider the basic linear AR(1) model (Ludlow, 2000).

$$x_t = \alpha x_{t-1} + \epsilon_t \tag{16}$$

Let x_t be a stationary random variable ϵ_t and be a white noise disturbance satisfying $E_{t-1}\epsilon_t^2 = E^2 = \sigma^2$ for every time period t. Equation (16) can be modified in various ways, such as incorporating deterministic regressors, introducing lagged values of $\{x_t\}$ to introduce moving average components, and including variables that explain the behavior of $\{x_t\}$. However, the essential characteristic of Equation (16) remains that the extent of autoregressive decay is determined solely by the constant value of α. It is feasible to estimate an ARMA model with time-varying coefficients without requiring prior specification of the adjustment process. The model exhibits linearity with respect to $\{x_t\}$, facilitating straightforward out-of-sample forecasts. Eq. (16) can be easily transformed into a random-coefficient model. Let's consider:

$$x_t = \alpha_t x_{t-1} + \epsilon_t \tag{17}$$

$$\alpha_t = a_0 + a_1\alpha_{t-1} + v_t \tag{18}$$

It is feasible to estimate the parameters a_0 and a_1 as well as the variances of both ϵ_t and v_t jointly. Due to the autoregressive coefficient at following the autoregressive process described by Eq (17), $\{x_t\}$, will exhibit periods characterized by relatively rapid and relatively slow autoregressive decay.

3.3.3.1. Approximating non-linear adjustment using Fourier series

A simple modification of Equation (16) involves allowing the autoregressive coefficient to vary with time, denoted by α_t. In contrast to Equation (18), we consider a deterministic but unknown function of time for the autoregressive coefficient [i.e., α_t]. The key difference between the random-coefficient model and our formulation is that we do not specify the exact form of $\alpha(\mathrm{t})$. However, under very mild conditions, the behavior of $\alpha(\mathrm{t})$ can be accurately represented using a sufficiently long Fourier series. For instance, if $\alpha(\mathrm{t})$ is an absolutely integrable function, it is possible to express it with any desired level of accuracy as follows:

$$y_t = \alpha(t)\, y_{t-1} + \epsilon_t \tag{19}$$

$$\alpha(\mathrm{t}) = \mathrm{A}_0 + \sum_{\mathrm{k}=1}^{\mathrm{s}} \left[\mathrm{A_k} \sin\frac{2\pi \mathrm{k}}{\mathrm{T}} * \mathrm{t} + \mathrm{B_k} \cos\frac{2\pi \mathrm{k}}{\mathrm{T}} * \mathrm{t} \right] \tag{20}$$

Here, the parameter s represents the number of frequencies present in the process that generates $\alpha(\mathrm{t})$. The crucial point is that the behavior of any deterministic sequence can be effectively captured using a sinusoidal function, even if the sequence itself is not periodic. Therefore, non-linear coefficients can be represented by a deterministic time-dependent coefficient model without initially specifying the nature of asymmetry and/or heteroskedasticity. This approximation approach is such that the standard ARMA model arises as a special case. In the event that the actual data-generating process is linear, all values of A and B in Equation (20) should be zero. Thus, instead of postulating a specific model, the focus shifts to selecting the appropriate frequencies to include in Equation (20). Since it is the process represented by the Fourier approach in (19) and (20), it is called the F-ARMA process. Given that s can be large, the estimation problem revolves around determining certain Fourier coefficients to include in the analysis.

3.3.3.2. Characteristics of first order F-ARMA models

The assumption underlying Equation (20) is that s is finite, allowing for the representation of $\alpha(\mathrm{t})$as a finite sum of Fourier coefficients. It is worth noting that the conventional AR(1) model arises as a special case when $\alpha(\mathrm{t}) = \mathrm{A}_0$. Additionally, it is assumed that $\alpha(\mathrm{t})$ is a deterministic bounded continuous function defined over the real numbers. Specifically, it is assumed to be a positive number L such that $0 < |\alpha(t)| < L < 1$for all integer values of t in the range [0, T]. Since $\alpha(\mathrm{t})$is known to be deterministic, the conditional mean and variance can be derived as follows:

$$E_{t-1} y_t = \alpha(t) y_{t-1} \tag{21}$$

$$E_{t-1} y_t^2 = \alpha(t)^2 y_{t-1}^2 + \sigma^2 \tag{22}$$

Therefore, given the information available in period (t-1), the persistence and volatility of the $\{x_t\}$ sequence are both positively associated with the absolute value of $\alpha(\mathrm{t})$ Since $\alpha(\mathrm{t})$ is not constant, the extent of autoregressive decay and the conditional variance will vary over time.

3.3.3.3. Methodological steps to use the Fourier-ARMA approach

To use the Fourier ARMA approach, it is necessary to follow the methodological steps below.
1. Determine the optimal ARMA model that provides the best fit, following the structure:

$$y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} + \sum_{i=1}^{p} \beta_i \epsilon_{t-i} + \epsilon_t \tag{23}$$

This can be accomplished either through the conventional Box-Jenkins methodology or by employing a straightforward procedure that entails selecting the model with the lowest SBC (Schwarz Bayesian Criterion). Regardless of the chosen estimation strategy, compute the SBC and store the residuals in the series $\{\hat{\epsilon}_t\}$. In the case of a linear process, Equation (21) should encompass all dynamic fluctuations in the $\{y_t\}$ series. However, in instances where non-linear behavior is suspected, identify the coefficient that is considered a particularly suitable candidate for temporal variability.

2. If coefficient "α_L" is chosen, estimate the following for each value of "k" within the range from 1 to T/2:

$$\hat{\epsilon}_t = A_k \sin\left(\frac{2\pi kt}{T}\right) y_{t-L} + B_k \cos\left(\frac{2\pi kt}{T}\right) y_{t-L} + \upsilon_t \tag{24}$$

Likewise, if coefficient "β_L" is selected, estimate the following for each value of "k" within the range from 1 to T/2

$$\hat{\epsilon}_t = A_k \sin\left(\frac{2\pi kt}{T}\right) \hat{\epsilon}_{t-L} + B_k \cos\left(\frac{2\pi kt}{T}\right) \hat{\epsilon}_{t-L} + \upsilon_t \tag{25}$$

If the inclusion of the most prominent frequency (k*) fails to decrease the SBC, conclude the search for significant frequencies and proceed to Step 4. Utilize a student's t-distribution to test the null hypotheses $A* = 0$ and $B* = 0$. If both exclusions are inconclusive, discontinue the search for significant frequencies and move on to Step 4.

3. If there are non-binding exclusion restrictions, impose them by setting $A*$ or $B* = 0$. Estimate the model by incorporating only the significant Fourier coefficient(s) while including frequency $k*$. For instance, if coefficient "α_L" is selected, proceed with the following estimation:

$$y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} + \sum_{i=1}^{n} \left[A_i^* * \sin\left(\frac{2\pi k_i^* t}{T}\right) + B_i^* * \cos\left(\frac{2\pi k_i^* t}{T}\right) + \quad \right] y_{t-L} + \sum_{i=1}^{q} \beta_i \epsilon_{t-i} + \epsilon_t \tag{26}$$

where 'k_i^*' represents the identified Fourier frequencies. Store the residuals as $\{\hat{\epsilon}_t\}$ and proceed back to Step 2.

$$y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} + \sum_{i=1}^{n} \left[A_i^* * \sin\left(\frac{2\pi k_i^* t}{T}\right) + B_i^* * \cos\left(\frac{2\pi k_i^* t}{T}\right) + \quad \right] y_{t-L} + \sum_{i=1}^{q} \beta_i \epsilon_{t-i} + \epsilon_t \tag{27}$$

where 'k_i^*' represents the identified Fourier frequencies. Store the residuals as $\{\hat{\epsilon}_t\}$ and proceed back to Step 2.

4. Once the identified frequencies and their associated non-zero values for $A*$ and/or $B*$ are available, the complete model estimation can be performed according to Equation (24), encompassing all the identified Fourier coefficients. It is crucial to conduct diagnostic checks at this stage. It has been observed that the inclusion of Fourier coefficients often leads to reduced p-values for various A_i^* and B_i^* terms. To determine whether any of these coefficients can be excluded from the model, one can utilize a standard t-distribution or F-distribution. Similarly, the t*-statistic can be employed to assess the significance of $A*$ and/or $B*$ values. If it is possible to exclude all values of $A*$ and/or $B*$, it can be concluded that the $\{y_t\}$ sequence does not exhibit any asymmetries.

3.3.4 TvAR Model

Autoregressive models utilize past values of the dependent variable as predictors to capture the patterns and progression of a process over time. This dependency follows a linear relationship, and the coefficients can be estimated using ordinary least squares (OLS) methodology. For a meaningful estimation of the coefficients, it is crucial that the variables in the model exhibit stationarity. The mathematical expression of the AR(p) model is represented by

$$y_t = \beta_0 + \beta_1 y_{t-1} + \cdots + \beta_p y_{t-p} + \gamma_1 x_{1t} + \cdots + \gamma_d x_{dt} + u_t \tag{28}$$

It is important to note that the dependent variable, yt, in equation (28), is influenced by its own lagged values and possibly other exogenous variables. The time-varying coefficient autoregressive (TvAR) model is a form of autoregressive model where the coefficients are functions of zt, which can represent a rescaled time period or a random process. For instance, in the TvAR(p) model, the regressors consist of the p-lagged values of the dependent variables.

$$y_t = \beta_0(z_t) + \beta_1(z_t)y_{t-1} + \cdots + \beta_p(z_t)y_{t-p} + \gamma_1(z_t)x_{1t} + \cdots + \gamma_d(z_t)x_{dt} + u_t \tag{29}$$

represents the specific mathematical expression of the model. The variable zt can be interpreted as the rescaled time, denoted as τ = t/T, or as a realization of a random variable. Notably, Chen and Tsay (1993) and Heydt et al. (2001) introduced the functional coefficient autoregressive (FAR) model, which is a type of TvAR model where the coefficients are functions of the lagged values of the dependent variable. In this FAR model,

$$y_t = \beta_0(y_{t-p}) + \beta_1(y_{t-p})X_{t-1} + \cdots + \beta_p(y_{t-p})X_{t-p} + u_t \tag{30}$$

represents the specific expression defining the functional coefficients. The parameters of the TvAR model are estimated using FLS, FLS-Kalman, and State-Space-Kalman algorithms (Kalaba & Tesfatsion, 1989).

4 Data, Estimation and Forecasting Results

The analysis utilized data for the USD/TRY exchange rate covering the period from January 2000 to December 2022. The data, consisting of a total of 276 observations, was obtained from TCMB EVDS. Descriptive statistics for the USD/TRY exchange rate are presented in Table 2, while the time series plot of its movement over the mentioned period is shown in Figure 1.

Table 2: Descriptive Statistics for USD/TRY Exchange Rate

	USD
Mean	3.246739
Median	1.685000
Maximum	18.65000
Minimum	0.550000
Std.Deviation	3.544271
Variation Coef.	109.1640
Skewness	2.677244
Kurtosis	10.40599
Jarque-Bera	960.4717
Probability	0.000000
Observations	276

During the analysis period, it is observed that the USD/TRY exchange rate has an average value of 3.25. The minimum and maximum values are 0.55 and 18.65, respectively. The positive skewness coefficient and the kurtosis value less than 3 indicate that the data of the USD/TRY series have a right-skewed and leptokurtic distribution compared to a normal distribution. These results suggest that the majority of the data points in the USD/TRY series are smaller than the mean, indicating a higher frequency of lower values, and also imply a large variance in the series. The very high coefficient of variation, such as 109.16, clearly indicates a substantial variance in the series. The Jarque-Bera test statistic and its probability value indicate that the USD/TRY series does not follow a normal distribution.

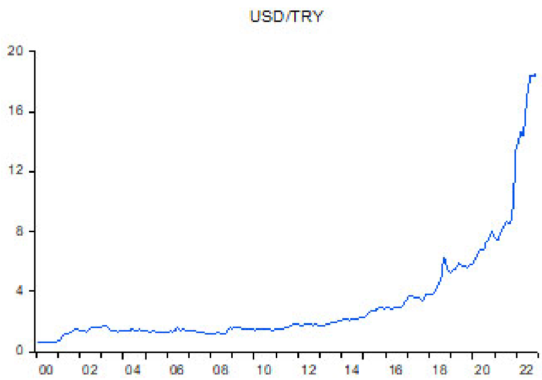

Figure 1: Time Series Plot of USD/TRY Exchange Rate

Analyzing the graph of the USD/TRY series, it becomes evident that there was a linear trend observed between the years 2000 and 2018. However, after 2018, it exhibits an exponential trend. Although the graph indicates non-linearity of the series, its linearity has been tested using the Harvey et al. (2007) and Harvey et al. (2008) tests, and the results are presented in Table 3.

Table 3: Linearity Tests

	Harvey (2007)			Harvey (2008)
Variable	W°10%	W°5%	W°1%	W_λ
USD/TRY	23.60***	23.67***	23.79***	12.31**

Note: ° indicate Harvey (2007) test. **, *** indicate %5, and %1 significance level respectively.

According to the information presented in Table 3, the test statistics for Harvey (2007) exceed the chi-square critical values with 4 degrees of freedom, and the test statistic for Harvey (2008) is greater than the chi-square critical value with 2 degrees of freedom at a significance level of 5%. Therefore, the null hypotheses stating that the series is linear are rejected. In other words, the USD/TRY series is not linear. The detection of stationarity for a nonlinear series is performed using unit root tests such as STAR, ESTAR or LSTAR.

Table 4: Sollis Test

Variables		Constant			Constant and Trend	
	Test stat.	F stat	lag	Test stat.	F stat	lag
USD	2.49688	4.32842	1	0.81816	1.51568	1
ΔUSD	46.0482***	35.1638***	1	46.0335***	41.7514***	1

Note: *** indicate %1 significance level respectively.

As a result of the examination of the information given in Table 4, it was concluded that the null hypothesis, which states that the hypothesis of the series is not stationary, is undeniable in the equation containing both constant and constant and trend for the level values of the USD variable. Since the USD series is not stationary, the first difference of the series was taken and then again subjected to the Sollis test. According to the test results applied to the first difference series, the null hypotheses are rejected because the test statistic and the F statistic are larger than the relevant critical values. That is, the series has ESTAR stationarity at the first difference. In addition, the series has an asymmetric feature in the mean reversion process. In other words, the rate of reversion to mean of negative and positive shocks is not the same.

Table 5: Flexible Fourier ADF Test

Variables	Constant				Constant and Trend			
	Test stat.	F stat	k	lag	Test stat.	F stat	k	lag
USD	-1.22744	13.38***	1	1	-3.93690	9.88***	1	1
ΔUSD	-11.8697***	4.65*	1	1	-11.8490***	3.74	1	1

Note: *, and *** indicate %10, and %1 significance level respectively.

Analyzing the data presented in Table 4, it was determined that the USD series' level values are non-stationary in both the equation with a constant term and the equation with both a constant and a trend term. In both equations, the test statistics are smaller than the critical values in absolute terms, indicating that the null hypothesis, which suggests that the USD series is non-stationary, cannot be rejected. Furthermore, when considering the equation with a constant term, it was determined that the F-statistic is not significant. This result indicates that there is no need for Fourier terms in the equation containing a constant term. However, in the equation containing both a constant and a trend term, since the F-statistic is greater than the critical value, the null hypothesis indicating the absence of frequency effect on the USD series is rejected. In other words, the frequency effect represented by sine and cosine on the USD series is significant. Since the USD series is non-stationary in the first difference, the series is subjected to the Flexible Fourier ADF test after taking the difference, and according to the obtained results, the test statistics are greater than the critical values in absolute terms, leading to the rejection of the null hypothesis suggesting that the USD series is non-stationary. In other words, the USD series is stationary in the first difference. Additionally, since the F-statistics are insignificant, it can be said that there is no frequency effect on the differenced series.

Based on the information presented in Table 4 and Table 5, it was concluded that the USD series is not stationary in level or it is an I(1) series. Thus, the first difference of the USD series will be used in the estimation phase of the ARIMA and Fourier-ARMA models. The aforementioned models were estimated and the results were presented in the following tables and figures.

According to the results provided in Panel A of Table 6, the AR and MA coefficients (except MA(2)) are statistically significant. The individual AR coefficients being individually and sum of the coefficients less than one in absolute value suggests that shocks in previous months do not have a lasting impact on the current month. Considering the MA coefficients, it can be observed that random shocks do not have a significant effect on the USD/TRY series. Based on the results in Panel A1, it is concluded that there is no autocorrelation in the residuals obtained from the ARIMA model, and the model is invertible. The ARFIMA results presented in Panel B indicate that $0<D=0.296224<0.5$ that the USD/TRY series exhibits long memory properties. In other words, the USD/TRY exchange rate is dependent on long past values, implying that there is an effect of long past on the current value. The individual and sum of AR coefficients being less than one suggest that past exchange rate changes do not have a permanent effect on the current exchange rate. Additionally, the significance of the MA coefficient being less than one indicates that random shocks have a temporary impact on the USD/TRY exchange rate. The results provided in Panel B1 demonstrate that the residuals obtained from the ARFIMA model do not exhibit autocorrelation and the model is invertible. Based on the results of both models, out-of-sample forecasts can be made for the USD/TRY exchange rate.

The estimation results of the F-ARMA model are provided in Table 7. Here, the frequency count is taken cumulatively as 24. According to the critical values of t and F shown in Table 1 by Ludlow (2000), it has been determined that the Fourier terms are statistically significant both individually and jointly. The statistical significance of Fourier terms indicates that in the F-ARMA model, either the constant coefficient or the long-term equilibrium value changes over time, which implies that the long-term value of the USD/TRY exchange rate is unstable. Since no autocorrelation problem was detected in the obtained residuals of the model, and the absolute values of the AR and MA parameters in the model are all less than one, the model is invertible. Therefore, this model can be used for out-of-sample forecasting.

Table 6: The Results of ARIMA and ARFIMA Models

Variables	Coefficient
Panel A: Results of ARIMA Model	
c	0.01055***
AR(1)	-0.49311***
AR(2)	0.44425***
AR(3)	0.84376***
MA(1)	0.95701***
MA(2)	-0.17543
MA(3)	-0.86454***
MA(4)	-0.41909***
Panel A1: Diagnostic check	
Breusch-Godfrey LM Test	Prob ($\chi^2(4)$)=0.9675
Stability Test	Model is invertible
Panel B: Results of ARFIMA Model	
c	1.592397
D	0.296224***
AR(1)	0.944991***
AR(2)	-0.54970***
AR(3)	0.304711***
MA(5)	-0.15728**
Panel B1: Diagnostic check	
Breusch-Godfrey LM Test	Prob ($\chi^2(4)$)=0.41
Stability Test	Model is invertible

Note: *, **, *** indicate %10, %5, %1 significance level respectively.

The prediction results of the time-varying autoregressive (TvAR) model are presented in Figure 2. Due to the fact that the coefficients in this model vary at every moment in time, it is difficult to present these results in a standard tabular format. Therefore, the TvAR model results are presented in graphical form. Additionally, in order to examine whether the coefficients are statistically significant over time, graphs depicting the time-varying t-statistics for each coefficient are provided. When examining the graph of the constant coefficient or the long-term equilibrium value associated with the model, it is observed that there is significant instability in the long-term equilibrium value of the USD/TRY exchange rate after 2008. Furthermore, the impact of the previous month's exchange rate changes on the current month's exchange rate has been found to be both negative (downward) and positive (upward).

Table 7: The Resultsfar of F-ARMA Model

Variable	Coefficient
C	0.012992***
AR(1)	0.955348***
AR(2)	-0.789373***
MA(1)	-0.872810***
MA(2)	0.685274***
SIN((2*3.14*1*t)/276)	-0.008479***
COS((2*3.14*1*t)/276)	0.011568***
SIN((2*3.14*2*t)/276)	-0.001011***
COS((2*3.14*2*t)/276)	0.009346***
SIN((2*3.14*3*t)/276)	0.002399***
COS((2*3.14*3*t)/276)	0.007372***
SIN((2*3.14*4*t)/276)	0.001780***
COS((2*3.14*4*t)/276)	0.003639***
SIN((2*3.14*5*t)/276)	0.002322***
COS((2*3.14*5*t)/276)	0.002543***
SIN((2*3.14*6*t)/276)	0.000534***
COS((2*3.14*6*t)/276)	-0.002805***
SIN((2*3.14*7*t)/276)	-0.003107***
COS((2*3.14*7*t)/276)	-0.008045***
SIN((2*3.14*8*t)/276)	0.001052***
COS((2*3.14*8*t)/276)	-0.001633***
SIN((2*3.14*9*t)/276)	0.001606***
COS((2*3.14*9*t)/276)	-0.008399***
SIN((2*3.14*10*t)/276)	-0.007665***
COS((2*3.14*10*t)/276)	-0.001948***
SIN((2*3.14*11*t)/276)	-0.000396*
COS((2*3.14*11*t)/276)	-0.002569***
SIN((2*3.14*12*t)/276)	-0.001787***
COS((2*3.14*12*t)/276)	-0.007730***
SIN((2*3.14*13*t)/276)	-0.002487***
COS((2*3.14*13*t)/276)	-0.011337***
SIN((2*3.14*14*t)/276)	0.004729***
COS((2*3.14*14*t)/276)	-0.001770***
SIN((2*3.14*15*t)/276)	-0.006052***
COS((2*3.14*15*t)/276)	0.004424***
SIN((2*3.14*16*t)/276)	-0.004064***
COS((2*3.14*16*t)/276)	0.006292***
SIN((2*3.14*17*t)/276)	0.001357**
COS((2*3.14*17*t)/276)	0.001869**
SIN((2*3.14*18*t)/276)	0.002297***
COS((2*3.14*18*t)/276)	0.009837***
SIN((2*3.14*19*t)/276)	0.007115***
COS((2*3.14*19*t)/276)	0.005584***
SIN((2*3.14*20*t)/276)	-0.000653
COS((2*3.14*20*t)/276)	0.004659***
SIN((2*3.14*21*t)/276)	0.003989***
COS((2*3.14*21*t)/276)	0.003385***
SIN((2*3.14*22*t)/276)	0.005642***
COS((2*3.14*22*t)/276)	-0.000626
SIN((2*3.14*23*t)/276)	0.002698**
COS((2*3.14*23*t)/276)	-0.002392*
SIN((2*3.14*24*t)/276)	0.004470***
COS((2*3.14*24*t)/276)	-0.004547***
Breusch-Godfrey LM Test	Prob (χ^2(**??**))=0.37**
Stability Test	Model is invertible

Note: *, **, *** indicate %10, %5, %1 significance level respectively.

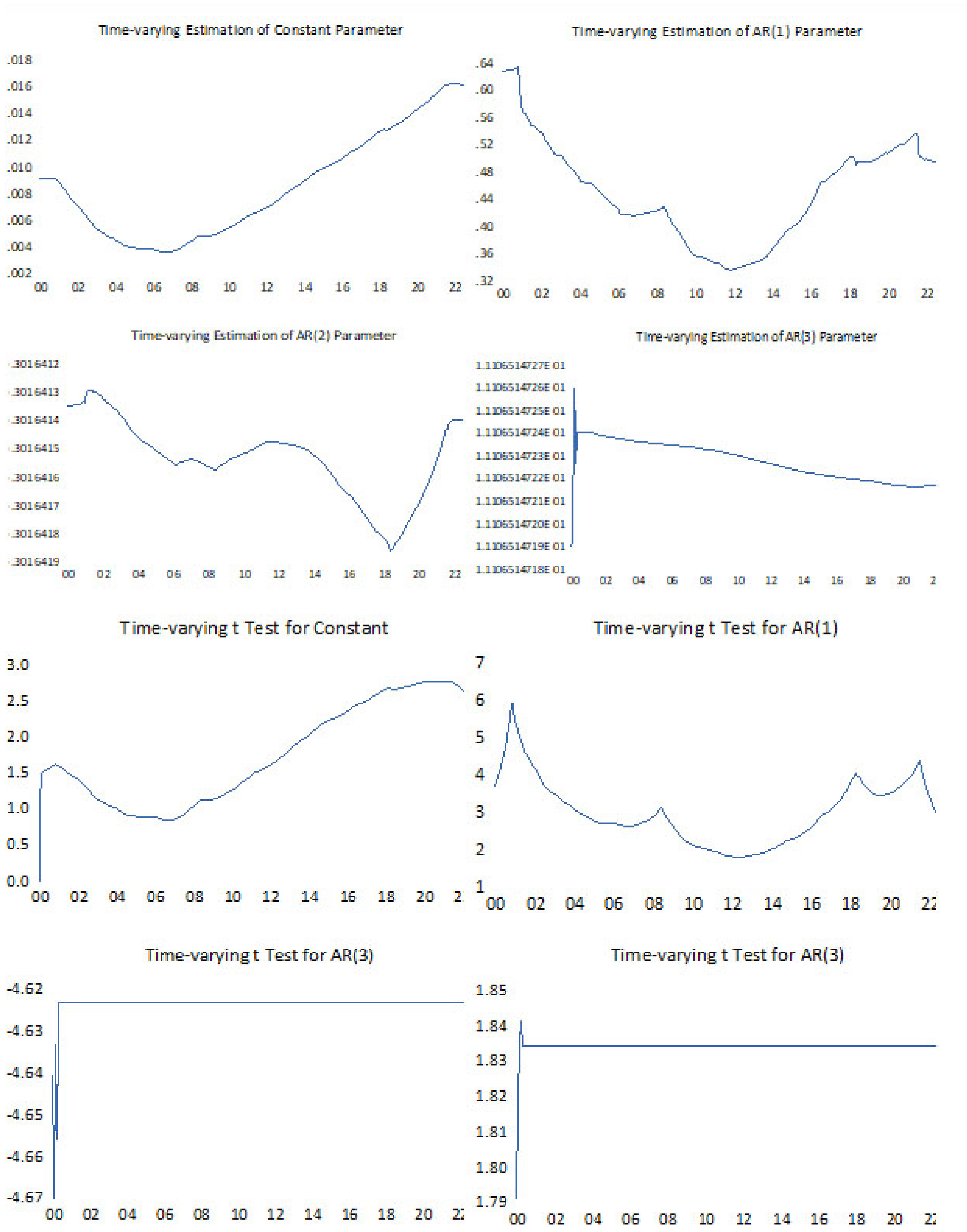

Figure 2: Time-varying AR Parametrs and t Tests

There is a continuous downward trend observed from January 2000 to September 2006, followed by an increasing trend between October 2006 and October 2008. A negative impact is observed again between November 2008 and April 2012, while a positive impact is observed between May 2012 and December 2022. The changes in the exchange rate two months ago have generally had a decreasing effect on the current period's exchange rate between January 2000 and October 2018, but had an increasing effect between November 2018 and December 2022. The impact of the changes in the exchange rate three months ago on the current exchange rate has remained relatively constant over time, indicating almost constant effect.

Table 8: Forecasting Comparison of the Models

Horizon	RMSE				MAPE			
	ARIMA	ARFIMA	F-ARMA	TvAR	ARIMA	ARFIMA	F-ARMA	TvAR
3	3.777053	4.45654	0.213243	0.19829	43.92516	70.05378	2.904676	2.08571
6	8.126489	8.72088	0.216847	0.20863	45.64104	73.83715	3.209128	2.68307
9	11.76294	10.3184	0.256209	0.21972	48.58427	79.51370	4.902713	2.96815
12	13.96528	13.6891	0.370841	0.26851	49.06719	84.00813	5.638144	3.68276

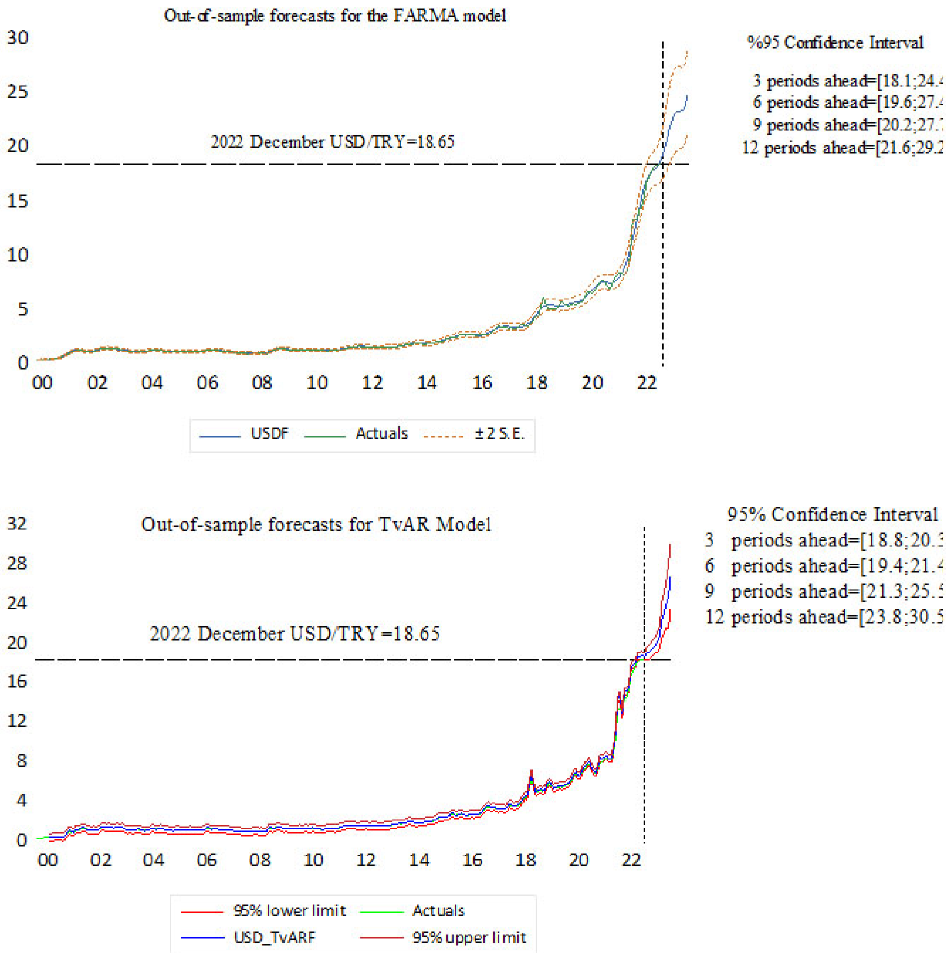

Figure 3: F-ARMA and TvAR

In order to comparing the performance of "in-sample" forecasts for the ARIMA, ARFIMA, F-ARMA, and TvAR models, the RMSE and MAPE criteria were used. The obtained results are presented in Table 8. According to the information given in Table 8, the F-ARMA and TvAR models exhibit better "in-sample" forecast performance. Comparing the forecast performances of the F-ARMA and TvAR models, it was found that the TvAR model outperforms the F-ARMA model. Therefore, out-of-sample forecasts were not conducted for the ARIMA and ARFIMA models. Out-of-sample forecast results using the F-ARMA and TvAR models are presented in Figure 3. As observed from the graphs, the confidence intervals calculated for the out-of-sample forecast values of the F-ARMA model are wider than TvAR. This result is due to the standard errors of the F-ARMA model being larger than those derived from the TvAR model. Considering these reasons, we can conclude that the out-of-sample forecasts of the TvAR model are more successful.

5 Conclusion

The main objective of this study was to provide evidence that the TvAR model would deliver superior forecasts compared to its alternative models, namely ARIMA, ARFIMA, and F-ARMA. To achieve this, linear tests conducted by Harvey (2007) and Harvey (2008) on the monthly USD/TRY series spanning 23 years revealed nonlinearity in the series. The series was found to be nonlinear based on the nonlinear Sollis (2009) unit root test and the flexible Fourier ADF (Enders-Lee, 2012) tests. Considering the forecasting performances of the ARIMA, ARFIMA, F-ARMA, and TvAR models estimated using the USD/TRY series, the TvAR model yielded better forecasting results.

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